

Policy, Planning, and Research

WORKING PAPERS

Development Economics

Office of the Vice President
Development Economics
The World Bank
June 1988
WPS 14

The Optimal Currency Composition of External Debt

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By choosing the appropriate currency composition of their external debts, developing countries can reduce the exposures associated with exchange rate, interest rate, and commodity price uncertainties.

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The increased volatility of exchange rates, interest rates, and goods prices has focused fresh attention on the importance for developing countries of reducing their risks in these markets. These countries generally cannot use such conventional hedging instruments as currency and commodity futures because of a variety of institutional and other constraints. But they can use the currency composition of their external debt to hedge against exchange rates and commodity prices.

The optimal currency composition of their foreign debt portfolios depends on the following factors:

- Domestic production structures.
- The shares of spending on different goods by consumers.
- The relationship between prices for domestic goods and exchange rates.

- The costs and risks of borrowing in foreign currencies.

- The effects of exchange rates on expected receipts and payments in foreign currencies.

What if a country only wants to hedge itself against the impact of commodity price and exchange rate movements and does not want to speculate on relative exchange rate movements? Its optimal currency composition will then hinge on the covariances between commodity prices and exchange rates, the covariances between exchange rates, and expected net foreign currency receipts.

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I. INTRODUCTION*

1. The increased volatility of exchange rates, interest rates and commodity prices over the last decade has affected many developing countries. It has at the same time highlighted the importance for developing countries of managing their currency, interest rate and commodity price exposure to reduce the risks faced in these markets. However, many developing countries have not been in a position to obtain access to the financial markets of the more developed countries--due to institutional and other considerations--to use hedging techniques which would reduce these exposures. Transfer of these risks to other market participants more able to absorb them or to transfer them further could have substantially benefitted developing countries.

2. It is not implied that developing countries will be unable to correct these shortcomings and manage their exposures in other ways if market access to the hedging techniques of the more developed country markets remains restricted. First of all, developing countries can engage in real diversification through the sourcing, producing and exporting of a mix of products which is close to optimal given the relationships between exchange rates and good prices. Secondly, the developing countries have a potential financial hedging instrument against unanticipated exchange rates movements in the form of the currency composition of their existing external liabilities.¹

* I am grateful to Stanley Fischer, Ron Duncan, Kathy Mann, Brian Pinto, and Dariush Malekpour for their comments.

¹ For instance, Kahn (1986) mentions that a number of rescheduling agreements between developing countries and commercial banks have provided currency-conversion options for non-U.S. banks. This type of development seems to indicate that developing countries have some room in rescheduling negotiations

In addition, developing countries can influence the currency composition of their new external financing. Thirdly, the currency composition of the external liabilities of developing countries can also be a useful tool to manage the exposure to commodity price and other goods price movements to the extent that movements in exchange rates and goods prices are correlated.

3. If one accepts the presumption that the currency composition of external liabilities can be an effective hedging instrument against movements in exchange rates and commodity prices (and net export receipts) several questions follow. First, what should be the overall objective function in choosing an optimal currency composition and what is the definition of risk that follows from the objective chosen? Second, what factors play a role in the optimal currency choice? Third, what practical rules to follow for the external liability management of a country? Fourth, what are the kinds of obstacles one can expect when one tries to implement the optimal currency composition model? Glaessner (1988) lists a fairly wide range of often intuitively appealing rules for determining the currency denomination of borrowings by a developing country that have been suggested by various authors. Glaessner criticizes most of these rules as not being explicitly related to a specific goal or objective and because "risk" has not been explicitly defined and measured. Similarly, Malekpour (1987) suggests that the decision rules proposed for the currency denomination of external debt of developing countries have been ad-hoc. Both authors conclude that a more integrated approach is necessary.

4. This paper uses findings from the literature on optimal portfolio

to alter the currency composition of their external debt to their own and commercial bank's interests.

theory to discuss the optimal currency composition of external debt and to answer the questions stated above. The analysis considers a small open economy facing a perfect world capital market and a large number of perfect commodity markets. The paper analyzes the optimal policy for the country as it seeks to maximize a social objective function, an intertemporal expected utility function defined over a country's consumption of the different goods. The paper starts out with a situation as general as possible, imposing few constraints on the production, price, and exchange rate processes and not unnecessarily restricting the objective function. Given the postulated processes for the exogenous variables, the optimal currency composition of the country's aggregate assets and external liabilities are derived. The general model allows identification of the factors which influence the optimal currency decision and thus directs attention to factors which have been ignored in previous approaches.

5. The drawback of the general model is that it does not necessarily provide clearcut policy rules with which to approach the problem of optimal currency denomination. The paper analyzes therefore a restricted version of the general model. The restricted version singles out the most important external exogenous variables affecting a country, exchange rates and commodity prices. This model results in closed-form solutions for the optimal currency shares and indicates the policy rules a country should follow in its external liability management. In the last part of the paper the necessary estimations and computations are described, including how to take into account the currency composition of existing external liabilities.

II. THE MODEL

Consumption and Investment Opportunity Sets

6. It is assumed that the domestic individual can buy K commodities. These K commodities can be classified into three groups: importables, exportables and nontradables. The time path of the price $P(i)$ of the i th commodity in terms of the domestic currency is described by a stochastic differential equation written as

$$(1) \quad \frac{dP(i)}{P(i)} = v(s,t)_{p(i)} dt + \sigma(s,t)_{p(i)} dZ_{p(i)}, \quad i=1, \dots, K$$

where s is an $S \times 1$ vector of state variables²; $v_{p(i)}$ is the instantaneous mean and $\sigma_{p(i)}$ is the instantaneous variance of the percentage rate of change of the price of the i th good; and $dZ_{p(i)}$ is a Wiener process. It is assumed that the individual takes the commodity prices as given. By assumption the state variables follow Ito processes.³ The first K state variables in the vector s are taken to be the logarithms of the prices of the commodities available in the domestic country. Some of the other state variables will be specified later.

² In the following, a vector will consistently be designated by a bold roman lower-case letter, whereas a matrix will be designated by a bold roman capital letter.

³ The properties of its processes and of the stochastic differential equations they follow are given in Merton (1971). See also Stulz (1981) (on which this part of the paper relies heavily) and Breeden (1979) for other references. The time paths of the prices of the commodities in terms of domestic currency can be related to the time paths of the prices of the commodities in terms of foreign currency (see further paragraph 8).

7. The country can denominate its liabilities and invest its wealth in N currencies (we exclude the currency of developing countries). It is assumed that each of the N exchange rates, $e(j)$, defined in terms of units of domestic currency per unit of foreign currency (e.g. Peso/\$), follow an equation similar to the equation which describes the price dynamics of the i th commodity,

$$(2) \quad \frac{de(j)}{e(j)} = v(s,t)_{e(j)} dt + \sigma(s,t)_{e(j)} dZ_{e(j)}, \quad j=1, \dots, N$$

where $v_{e(j)}$ is the instantaneous mean (i.e., the expected rate of depreciation of the home currency versus the foreign currency j), and $\sigma_{e(j)}$ is the instantaneous variance of the percentage change of the j th exchange rate. The last N state variables in the state vector s are taken to be the logarithms of the exchange rates. The remaining state variables remain unspecified, but could include the total market values of the domestic or foreign assets in which the investor can invest and the domestic and foreign money supplies.

8. It is not assumed that the law of one price holds exactly for all traded goods, i.e., $P(i) \neq e(j)P^*(i,j)$, for all i and j , where $P^*(i,j)$ is the price of the traded good i in terms of the foreign currency j . Due to trade barriers, oligopolistic pricing, transaction costs and/or barriers to international commodity arbitrage, the law of one price does not hold at all points in time. In contrast to the models of Krugman (1981), de Macedo (1982) and Stulz (1983), at this point we do not assume either that changes in the terms of trade are perfectly correlated with the (weighted average of the) changes in the exchange rates. This would be the case if we were to assume that domestic prices were perfectly sticky. We leave thus the precise

mechanism through which nominal exchange rate movements influence the prices of exportable, importable and nontradable goods unspecified. To try to explain the effects of currency movements on the behavior of absolute and relative prices would require the development of a much more elaborate model than we are prepared to undertake here {see Giovannini (1986), Dornbusch (1987), and Varangis and Duncan (1987), for some facts and explanations on international price relations and further references}. We later on take as given the variance-covariance matrix of the prices and the exchange rates.

9. All domestic assets are assumed to be traded. Goods are produced by profit-maximizing firms whose shares are held by domestic investors. The production technologies are assumed to be constant returns to scale. Domestic investors can invest in (or borrow) the N foreign currency bonds. However, due to certain barriers, domestic investors are prevented from investing in foreign stocks.⁴ It is assumed that: (1) for each asset the returns only accrue in the form of capital gains; (2) there are no transaction costs; (3) unlimited short sales with full use of proceeds is permitted; and (4) markets are always in equilibrium.

10. The investor can invest in L domestic risky assets. Let $I(i)$ be the price in domestic money of a domestic risky asset. It is assumed that $I(i)$ follows a stochastic differential equation,

$$(3) \quad \frac{dI(i)}{I(i)} = v(s,t)_{I(i)} dt + \sigma(s,t)_{I(i)} dZ_{I(i)}, \quad i=1, \dots, L$$

⁴ This restriction prevents domestic investors from using the foreign stocks as hedging instruments against unanticipated commodity price or exchange rate changes, or as hedges against unanticipated changes in the other state variables.

where $v_{I(i)}$ is the instantaneous expected rate of change of the price of the i th asset and $\sigma_{I(i)}$ is the instantaneous standard deviation of the rate of change of the price of asset i . Both $v_{I(i)}$ and $\sigma_{I(i)}$ can be functions of the state variables s . It is assumed that there exists in each country, foreign as well as domestic, one nominal riskless bond. Let $B^*(j)$ (B) be the price in the j th foreign (domestic) currency of the foreign (domestic) nominal safe asset. The dynamics for $B^*(j)$ are given by

$$(4) \quad \frac{dB^*(j)}{B^*(j)} = R^*(j)dt, \quad j = 1, \dots, N$$

where $R^*(j)$ is simply the instantaneous nominal rate of return on a safe foreign bond in currency j . Similarly, R is the instantaneous nominal return on a safe domestic bond.

Properties of Asset Demand Functions

11. Define the excess return of the j th foreign bond for a domestic investor, written $dH(B^*(j))/H(B^*(j))$, as the return on one unit of domestic currency invested in the foreign bond, financed by borrowing at the interest rate R in the domestic country, i.e.,⁵

$$(5) \quad \frac{dH(B^*(j))}{H(B^*(j))} = R^*(j)dt + \frac{de(j)}{e(j)} - Rdt, \quad j=1, \dots, N$$

$$= (R^*(j) + v_{e(j)} - R)dt + \sigma_{e(j)}dZ_{e(j)}$$

⁵ It is not assumed that uncovered interest rate parity holds. In other words, $R^*(j) + v_{e(j)} - R$ is not necessarily equal to zero.

This equation implies that the excess return on an instantaneous foreign safe nominal bond $B^*(j)$ is perfectly correlated with the change in the corresponding exchange rate, $e(j)$.⁶

12. In the following, the return on an investment of one unit of domestic currency in the domestic risky asset $I(i)$ by a domestic investor, financed by borrowing in the home-country, is called the excess return on the i th domestic asset, i.e.

$$(6) \quad \frac{dH(I(i))}{H(I(i))} = \frac{dI(i)}{I(i)} - Rdt, \quad i=1, \dots, L$$

$$= (v_{I(i)} - R)dt + \sigma_{I(i)} dZ_{I(i)}$$

Let v be the $(L+N) \times 1$ vector of the expected excess returns on risky investments for domestic investors. The L -first elements of v comprise the expected excess return on the L domestic risky assets, whereas the last N elements comprise the expected excess returns on the N foreign bonds.

13. It is assumed that the domestic investor maximizes a lifetime expected utility function which is von Neuman-Morgenstern, time-additive and depends only on the consumption of the K commodities and time. The expected

⁶ The foreign bond $B^*(j)$ is of an instantaneous maturity and has thus only exchange risk and no interest risk. The domestic currency returns on long-term foreign bonds are influenced not only by the exchange rate movements but also by foreign interest movements which lead to foreign capital losses and gains. This could reduce the correlation between the excess return on long-term foreign bonds and the exchange rate to less than 1 and could make long-term foreign bonds a less useful hedging instrument against movements in the corresponding exchange rate. Adler and Simon (1986) show, however, that the US\$-returns on foreign long-term bonds were essentially 100% exposed to the respective exchange rate and behave in that respect very much like foreign short-term deposits.

utility function of the investor is given by:

$$(7) \quad E_t \left\{ \int_t^\infty U(c_1(s), \dots, c_k(s)) e^{-\delta s} ds \right\}$$

where E_t is the expectation operator conditional on all information available at time t . $c_i(s)$ is the consumption rate of good i and δ is the intertemporal rate of time preference.

14. Let $[w:b]'$ be the investments in risky assets of the domestic investor, expressed as a fraction of his wealth, where w is a $1 \times L$ vector whose representative element w_i is the proportion of the wealth the domestic investor invests in the i th domestic risky asset and where b is a $1 \times N$ vector whose representative element b_j is the proportion of the investor's wealth invested in the j th foreign currency bond. The remaining fraction of the investor's wealth, that part that is not invested in domestic risky assets or foreign bonds, is invested in the domestic safe bond. Finally, define V_{aa} as the $(L+N) \times (L+N)$ variance-covariance matrix of the excess returns and V_{as} as the variance-covariance matrix of those excess returns with the changes in the state variables, the first K state variables being the logarithms of the commodity prices the investor faces and the last N state variables being the logarithms of the exchange rates.

15. The optimization problem the investor faces is now formally equivalent to the problem faced by the investor in Stulz (1981) and Breeden (1979). The optimal portfolio for the domestic investor is characterized by the following investments in risky assets.

$$(8) \quad [w:b]' W = \left(\frac{T^a}{C_w} \right) V_{aa}^{-1} v + V_{aa}^{-1} V_{as} \left\{ \frac{-C_s}{C_w} + \left(\frac{C_a}{C_w} - \frac{T^a}{C_w} \right) \right\}$$

where $C(W,s,t)$ is the consumption expenditure function of the investor. By convention C is equal to $P'c$, where c is the $K \times 1$ vector of the rates at which each commodity i is consumed by the investor. The partial derivatives of the consumption expenditure function with respect to the state variables is the $S \times 1$ vector C_s and the partial derivative of the consumption expenditure function with respect to wealth is C_w . α is the $K \times 1$ vector of average expenditure shares of the K goods available in the domestic country, i.e., $P_i c_i / C = \alpha_i$. m is the $K \times 1$ vector of the marginal expenditure shares of the same goods, i.e., $P_i (\partial c_i / \partial C) = m_i$. Finally, T^a is the absolute risk tolerance coefficient of the domestic investor, i.e., $T^a = -U_c / U_{cc}$, where $U(C(W,s,t), P, t)$ is the investor's indirect utility function of consumption expenditures when the prices of available commodities are given by the $K \times 1$ vector P . T^a is in other words the inverse of the absolute risk aversion coefficient. 0 is here an $(S-K) \times 1$ vector of zeros.

16. Equation (8) can be interpreted as a weighted sum of $(S+1)$ column vectors, such that the vectors do not depend on investor's tastes and preferences whereas the weights do. The column vectors, appropriate scaled, can be interpreted as mutual fund portfolios. The first mutual fund portfolio is a mean-variance efficient portfolio (i.e. there is no portfolio which dominates this portfolio by having a higher mean for the same variance or a lower variance for the same mean) and is given by the $(L+N) \times 1$ vector $[w^m; b^m]'W = (T^a / C_w) V_{aa}^{-1} v$. The other S mutual funds are hedging portfolios and are given by the S column vectors, each of dimension $(L+N) \times 1$, of the matrix resulting from the product $V_{aa}^{-1} V_{as}$. These S mutual funds are used by investors to hedge against unanticipated changes in the state variables. The paper discusses the

economic implications of the model in the next section with a focus on the hedging portfolios.

III. IMPLICATIONS OF THE MODEL FOR THE OPTIMAL CURRENCY COMPOSITION OF EXTERNAL DEBT

17. The domestic investor's holdings (long or short) of foreign bonds is represented by the vector b . The demand for foreign bonds can be decomposed into two parts. First, the investor has a "speculative" demand for foreign bonds which does not depend on his consumption preferences and is given by his holdings in foreign bonds of the mean-variance portfolio b^m times his wealth, i.e. $b^m W$. It is the demand for foreign bonds in the portfolio which corresponds to the point of tangency of the capital market line and the efficient frontier of portfolios of risky assets in the space of nominal returns and standard deviation of those returns. The absolute value of these holdings will increase with the relative level of risk tolerance and thus decrease with the relative level of risk aversion. The investor will make a speculative investment in a foreign bond $B^*(j)$ (positive or negative) whenever its expected excess return, $R^*(j) - v_e(j) - R$, is different from zero and/or when the exchange rates are correlated with the domestic asset returns. Second, the investor holds foreign bonds because the returns on these bonds are correlated with the changes in the state variables: the K commodity prices, the other (not specified) state variables, and the N exchange rates.

18. The demand for foreign bonds, explained by the fact that foreign bonds are hedges against the changes in the state variables, is given by:

$$(9) \quad [0:b^h]' W = [0:1]' v_{aa}^{-1} v_{as} \left\{ -\frac{C_s}{C_w} + \left(\frac{C_a}{C_w} - \frac{T^a}{C_w^m} \right) \right\}$$

0

where, as stated, C_s is the $S \times 1$ vector of partial derivatives of the consumption function with respect to the logarithms of the S state variables. The 0 vectors are respectively two $1 \times L$ and a $1 \times (S-N)$ vector of zeros and 1 is a $1 \times N$ vector of ones. As said, we can interpret $V_{aa}^{-1} V_{as}$ as the matrix of the composition of the S mutual fund portfolios, which have the highest correlation with the state variables, in terms of shares of domestic and foreign risky assets. We can write C_s' as $[C_p; C_s; C_e]'$, where C_p is the vector of partial derivatives of the consumption function with respect to the logarithms of the K commodity prices; C_s , idem with respect to the non-specified state variables s' ; and C_e idem with respect to the logarithms of the N exchange rates. Now decompose the hedging demand for foreign bonds into its three components, each of which is due to the fact that foreign bonds can hedge the investor against changes in one of the following groups of state variables: (1) the commodity prices; (2) the not further specified state variables; and (3) the exchange rates.

19. The hedging demand for foreign bonds due to unanticipated changes in commodity prices is the most complicated. We can decompose the effects of an unanticipated change in a commodity price on the marginal utility of consumption expenditures. Firstly, for a constant amount of consumption expenditures, a change in a commodity price affects the marginal utility of consumption expenditures because it affects the quantities the investor can consume for these expenditures. Secondly, a change in a commodity price, for a given value of the investor's wealth, affects the amount of consumption expenditures itself, as the investor rearranges his lifetime expenditure plans. Stulz (1983) calls these effects respectively price-level effects and intertemporal substitution effects.

20. The price-level effect is given by $(C_\alpha/C_w - (T^a/C_w)m)$ and the intertemporal substitution effect is captured by the vector C_p/C_w (part of the vector C_g/C_w). We first analyze the intertemporal substitution effect. The vector C_p captures the desires of the investor to use mutual funds, partially consisting of foreign bonds, as a hedge against those unanticipated changes in the investment and consumption opportunity sets which are associated with commodity price changes. The domestic return on a foreign bond j is perfectly correlated with exchange rate j and a foreign bond can thus be used to hedge perfectly against changes in exchange rates. Changes in the exchange rates are likely to be correlated with changes in the commodity prices. Accordingly, the foreign bonds in the mutual fund portfolios can be used to hedge the investor partially against those unanticipated changes in commodity prices which are associated with unanticipated changes in the investment and consumption opportunity sets. If the indirect utility function of consumption expenditures exhibits constant relative risk aversion, $C_g = 0$, the investor will not want to hedge against intertemporal substitution effects. Alternatively, assuming that the distribution of the expected excess returns on assets and the distribution of the commodity prices are constant, together with the assumption of a constant real income, also implies that $C_g = 0$.

21. The price-level effect can be rewritten, as done by Stulz (1983), as $(1 - T^r \epsilon) \alpha C/C_w$, where ϵ is the $K \times 1$ vector of the consumption expenditure elasticities of the K goods, i.e., $\epsilon_i = (\partial c_i / \partial C) * (C/c_i)$, and T^r is the relative risk tolerance of the investor's indirect utility function of consumption expenditures. For an investor with infinite relative risk aversion (a relative risk tolerance of 0) the demand, due to the price-level effect, for the mutual fund portfolios that have the highest correlation with the

commodity prices (the first K vectors in of $V_{aa}^{-1} \times V_{as}$) is explained by the vector of expenditures shares α . Such an investor would own the mutual fund portfolios, which have the highest correlation with commodity prices, for amounts of $\alpha C/C_w$. After an unanticipated change in commodity prices the investor will be able to consume "as near as possible" the basket of commodities he would have consumed without that change. The qualifier, "as near as possible", depends on the degree of correlation between the changes in the commodity prices and the changes in the prices of the mutual funds.

22. The remaining demand for the "commodity price" mutual funds, due to the price-level effect is $-T^R \epsilon \alpha C/C_w$, and can be explained by the fact that the investor is willing to bear some commodity price risk, if, by doing so, he can increase his expected consumption at future dates. The higher the risk tolerance, the more he is willing to enter gambles which involve price uncertainty and have a positive expected real excess return. The gamble concerns investing less than his expenditure shares would call for in each of the "commodity price" mutual funds.

23. It follows thus that the total demand for the "commodity price" mutual funds, explained by the fact that the investor consumes goods, is:

- i) an increasing function of the risk aversion of the investor.
- ii) an increasing function of the average consumption expenditure shares if $T^R \epsilon_i \leq 1$ and a decreasing function otherwise.
- iii) a decreasing function of the marginal budget shares.

The necessary condition for being long in each of the K "commodity price" mutual funds, as a result of the price-level effect alone, is $T^R \epsilon_i < 1$, as noted first by Stulz (1983).

24. The vectors C_s , and C_e (part of the vector C_s/C_w) indicate the demand

of the investor for the mutual fund portfolios that provide a hedge against the unanticipated changes in the (S-K-N) nonspecified state variables and the N exchange rates. As we saw above, the return on the foreign bond $B^*(j)$ is perfectly correlated with the changes in the exchange rate $e(j)$. Accordingly, if the return on a foreign bond (in other words, the changes in an exchange rate) is correlated with the unanticipated changes in the nonspecified state variables then the foreign bond can provide a hedge against unanticipated changes in these state variables which are associated with unanticipated changes in the investment and consumption opportunity sets. Similarly, a foreign bond can provide a hedge against unanticipated changes in the exchange rate that are associated with unanticipated changes in the investment and consumption opportunity sets. We can translate this into a vector of demands for foreign bonds using the exact composition of the hedge portfolios.

25. Analyzing the different components of the demand of the investor for the mutual funds does not tell us the final amount invested or borrowed by the investor in each of the N currencies unless we know the composition of the S mutual funds as far as the shares of each of the N foreign bonds is concerned. Investors have limited hedging possibilities because the quality of the hedge depends on the correlation between the unanticipated changes in the prices of the mutual funds and the unanticipated changes in the state variables which is not perfect in this model.

IV. SPECIFIC EXAMPLES

26. The model outlined above is very general. The analysis indicates that the following major factors should influence the currency denomination of external debt: 1) the covariances of exchange rates, 2) the covariances of commodity prices and exchange rates, 3) the consumption and production structures of the economy, 4) the expected costs of borrowing in each of the foreign currencies and the covariances among these cost of borrowings, 5) the level of risk tolerance in the economy. However, the general model does not provide a closed-form solution nor straightforward policy implications and practical decision rules for the currency composition of external liabilities. Without (i) specifying exactly what the state variables are, (ii) postulating their stochastic behavior, and (iii) further restricting the utility function, no definite conclusions can be reached regarding the optimal currency composition of external debt. We, therefore, use a specific version of the model above to derive the optimal currency denomination of external debt and demonstrate explicitly some of the ideas outlined above. The restricted model is similar to one proposed by Malekpour (1987), and has some of the features of the models of Krugman (1981), Stulz (1984) and Fraga (1986).

27. The investor can invest (or borrow) in N foreign currencies bonds, $B^*(j)$, with a fixed foreign nominal interest $R^*(j)$. The investor can also invest in a domestic safe bond (B) whose nominal return is fixed at R . The postulated processes for the N exchange rates, $e(j)$, defined in terms of domestic currency per unit of foreign currency, are lognormal,

$$(10) \quad \frac{de(j)}{e(j)} = \mu_j dt + \sigma_j dZ_e(j) \quad j = 1, \dots, N$$

where μ_j is the expected rate of change of currency j respectively; σ_j is the standard deviation of the rate of change of currency j ; $dZ_{e(j)}$ is the increment of a Wiener process. The covariance coefficient between the rates of change of N exchange rates is σ_{ij} .

28. As a result of exports, the country will receive at time T payments in each of the foreign currency $e(j)$ equal to $F^*(j)$, $j = 1, \dots, N$. The current value in terms of the foreign currency $e(j)$ of each of these amounts is $F(j)$ and the current value in terms of domestic currency of each of these payments is thus $e(j)F(j)$.⁷ The current values in foreign currency of the future payments follow standard lognormal processes given by,

$$(11) \quad \frac{dF(j)}{F(j)} = \eta_j dt + \zeta_j dZ_{f(j)} \quad j = 1, \dots, N$$

where η_j is the expected rate of change and ζ_j is the standard deviation of the rate of change of the current value in foreign currency j of the payment at time T . Using Ito's lemma we can derive the process for the domestic current value of the payment at time T in currency j as,

$$(12) \quad \frac{d(e(j)F(j))}{e(j)F(j)} = (\mu_j + \eta_j + \sigma_{fj})dt + \sigma_j dZ_{e(j)} + \zeta_j dZ_{f(j)}$$

where σ_{fj} is the covariance between the rate of change in the exchange rate j and the current value in foreign currency j of the payment at time T .

⁷ The exact currency in which the amounts are received is immaterial to our results; what matters is that the future amounts have stochastic current values in terms of the domestic currency.

29. We ignore for the moment the domestic tradable assets in the form of the domestic investment opportunity set $I(i)$ and assume that the investor can only invest in domestic and foreign currency bonds. Furthermore, we restrict the process for the K commodities the investors consumes to be lognormal. The postulated processes are thus

$$(13) \quad \frac{dP_i}{P_i} = v_i dt + \tau_i dZ_{P(i)} \quad i = 1, \dots, K$$

v_i is the expected rates of change of price i and τ_i is the standard deviations of the rate of change of price i ; $dZ_{P(i)}$ is the increment of a Wiener processes. The covariance coefficient between the rates of change of two prices is τ_{ij} . We specify the covariance between the logarithms of the prices and the N exchange rates to be λ_{ij} , where $i = 1, \dots, K$, and $j = 1, \dots, N$. We do not (have to) make the assumption that Purchasing Power Parity holds nor that changes in the terms of trade are perfectly correlated with changes in exchange rates.

30. We limit the time horizon to the date T . It is assumed that the investor maximizes his or her lifetime expected utility function over the consumption of the K commodities. The objective function is of the form,

$$(14) \quad E_t \left\{ \int_t^T \frac{1}{\gamma} \{c(1)^{\alpha(i)}, \dots, c(K)^{\alpha(k)}\}^\gamma e^{-\delta s} ds \right\}$$

where $c(i)$ is the rate of consumption of the good i . The rate of time preference, δ , is constant. The utility function assumed exhibits constant relative risk aversion, i.e., $1-\gamma$ is $-U''C/U'$ ($\gamma < 1$). In addition, the utility function is characterized by constant expenditure shares; the investor

will always spend a share $\alpha(i)$ of his total expenditures on good i .

31. The model must satisfy a budget flow constraint. Making use of the compact notation developed for the general model, we can write the behavior of nominal wealth as

$$(15) \quad dW = \sum_{j=1}^N b(j) \frac{dH(B^*(j))}{H(B^*(j))} W + \{RW - \sum_{j=1}^K P(j)C(j) + \sum_{j=1}^N e(j)F(j)(\mu_j + \eta_j + \sigma_{fj})\}dt + \sum_{j=1}^N e(j)F(j)(\sigma_j dZ_e(j) + \tau_j dZ_f(j))$$

where W is the investor's nominal wealth, $dH(B^*(j))/H(B^*(j))$ are the excess returns on the foreign bonds compared to the domestic safe asset measured in domestic currency, and $b(j)$ are the shares of wealth invested in the foreign bond or borrowed in foreign currency j . The equation for the behavior of nominal wealth indicates the dependence of nominal wealth on the changes in the current domestic currency values of the future payments in foreign currencies. The investor will choose its composition of foreign bonds taking into account the stochastic behavior of the domestic currency values of the foreign currency payments. In addition, the investor will hedge himself against the unanticipated movements in the prices of the K goods because of price-level effects. The investor's only opportunity to hedge for price effects is by investing in or borrowing the only assets whose returns may be correlated with changes in the prices, the N foreign bonds.

32. We can solve for the shares of wealth invested in foreign bonds but need to introduce some notation first. v stands for the $N \times 1$ vector of excess returns on foreign bonds. V_{ee} ($N \times N$) denotes the variance-covariance matrix of the excess returns on the N foreign bonds which, given that these excess returns are perfectly correlated with the exchange rates, is nothing else but

the variance-covariance matrix of the exchange rate movements. V_{es} ($N \times (N+K)$) denotes the variance-covariance matrix of the excess returns (i.e., of the changes in the exchange rates) with the logarithms of K exogenous prices and the N exchange rates. V_{ef} ($N \times N$) denotes the variance-covariance matrix of the changes in the exchange rates with the changes in the current domestic currency values of the future receipts of foreign currencies. The solution for the optimal amounts invested in foreign bonds or borrowed in foreign currency is as follows,

$$(16) \quad bW = \frac{1}{1-\gamma} V_{ee}^{-1} v[W] - V_{ee}^{-1} V_{ef}[ef] + (1 - \frac{1}{1-\gamma}) V_{ee}^{-1} V_{es} \begin{pmatrix} a \\ 0 \end{pmatrix} [W]$$

where 0 is a $N \times 1$ vector of zeros and where ef is the vector of the domestic currency equivalents of the current foreign currency values of the future payments.⁸

33. Equation (16) implies that the demand for foreign bonds consist of $S+1=N+K+1$ mutual funds. The first fund is the mean variance (speculative) portfolio, $b^m W = (1/(1-\gamma)) V_{ee}^{-1} v[W]$. The speculative demand for foreign bonds depends on the expected return excess on the bonds and the inverse of the variance-covariance matrix of exchange rates. The higher the risk aversion coefficient, $1-\gamma$, the lower the investment will be in this mutual fund. The minimum variance hedge is $-V_{ee}^{-1} V_{ef}[ef]$ and is independent of the level of risk aversion. The investor will borrow in foreign bonds as they can insulate him against changes in the domestic currency value of the future payments. These domestic currency value changes are caused not only by movements in the

⁸ See Stulz (1984), and Adler and Detemple (1988) for a similar equation.

exchange rate at which the payment is converted into domestic currency, but also by movements in the cross-currencies which change the current foreign currency value of the foreign payment itself.⁹ The price-level hedges, due to the fact that the exchange rates covary with the prices of the K commodities, are dependent on the vector of consumption shares α . The demand for the price-level mutual fund portfolios increases with the level of risk aversion and is positive when $\gamma < 0$, which is equivalent to a relative risk aversion coefficient larger than 1.

34. Things will be even simpler if we assume that there exists only one expected payment of foreign currency and only one foreign bond to hedge earnings and relative price risk. We can then write the demand for this bond as:

$$(17) \quad bW = \frac{1}{1-\gamma} \left(\frac{R^* + v - R}{\sigma_e^2} \right) [W] - \frac{(\sigma_e^2 + \sigma_{ef})}{\sigma_e^2} [ef] + \left(1 - \frac{1}{1-\gamma} \right) \frac{1}{2} v_{es} \left(\frac{\alpha}{0} \right) [W]$$

The minimum variance hedge, $b^m W = -([\sigma_{ef} + \sigma_e^2]/\sigma_e^2) eF$, reduces to holding a short position (borrowing) in the foreign currency for an amount equal to the current value in foreign currency of the expected payment when the current value in foreign currency of the expected payment is uncorrelated with the

⁹ The minimum-variance hedge could thus capture the desire to hedge against the changes in the structure of the earnings the country receives for a particular commodity due to cross-currency changes. Changes in cross-currency rates among developed countries can drastically affect the relative market share of a developing country even if the exchange rates between the developing country and those developed countries to which it exports remain constant. As an example, if a country has a pegged exchange rate with the yen and exports only to Japan, cross-currency changes can still affect its export earnings from Japan as other market participants become more or less competitive.

changes in the exchange rate ($\sigma_{ef} = 0$).

35. It is useful to look at the price level hedge in two polar cases that corresponds to well-known models of exchange rate determination. In the first polar case, it is assumed that Purchasing Power Parity (PPP) holds. If P^* denotes the foreign price level and P denotes the domestic price level, where $P = \Pi P(i)^{\alpha(i)}$, then $P^*e = P$. This implies that the demand for the foreign bond, due to the desire to hedge relative price movements, can be simplified to, $b^{PW} = [1 - (1/(1-\gamma))] [(\sigma_P^2 - \sigma_{PP}^*) / (\sigma_P^2 + \sigma_P^{*2} + 2\sigma_{PP}^*)] [W]$. The minimum variance hedge remains the same. In the case that there exists no unanticipated foreign inflation the foreign bond is a perfect hedge against unanticipated domestic inflation. If there is only foreign and no domestic unanticipated inflation the investor does not need to hedge for the price level effect.

36. The second polar case is when exchange rate changes are perfectly correlated with changes in the terms of trade. In this setting, which can be modelled by assuming that the prices of goods are constant in terms of the currency of the country in which they are produced and that the law of one price holds, changes in the prices of imported goods are perfectly correlated with the exchange rate. It follows then that the price level hedge is given by $[1 - 1/(1-\gamma)] \alpha(f) [W]$, where $\alpha(f)$ are the expenditure shares of the imported goods. The foreign bond provides a perfect hedge against unanticipated changes in the prices of foreign good.

37. The relative weights of the mean-variance and the price-level portfolios depend on the relative risk aversion coefficient. Empirical evidence exists for developed countries which indicates that the relative risk aversion coefficient is larger than one. For our purpose it seems more than reasonable to assume, taking into account the low income levels in developing

countries, that the relative risk aversion coefficient is above one. This implies that $\gamma < 0$. The other factors that influence the demand for foreign bonds will have to be obtained by estimation or computation. We discuss the techniques to do this in the next sections.

38. The analysis above ignored the production structure of the domestic economy. Including a production process will, however, not change the basic structure of the optimal liability composition. This is most easily demonstrated using a two-period framework along the lines of Benninga, Eldor and Zilcha (1985). A risk averse firm, producing solely for the export market, decides in time 0 on the amount of its domestic wealth to be invested for the production at time 1 of a commodity whose domestic currency price in period 1, P , is uncertain as of time 0. The uncertainty in the domestic currency price of the commodity is a reflection of both the uncertain commodity price in terms of foreign currency and the uncertain exchange rate. Call the amount invested in the production process X . In addition, the firm has the possibility to invest in (or sell short) a one-period domestic bond, B , and N one-period foreign currency bonds, $B^*(j)$, $j = 1, \dots, N$. The firm also receives an exogenous income per period of Y . The firm maximizes the expected value of an objective function over its income at time 0, I_0 , and time 1, I_1 , of the following form,

$$(18) \quad \max U(I_0) + \delta U(I_1)$$

The objective function $U(I)$ is assumed to be concave, increasing and continuous. δ is a fixed factor of time-preference, $0 \leq \delta \leq 1$.

39. Time zero net income for the firm is given by,

$$(19) \quad I_0 = \bar{Y} - W - B - \sum_{j=1}^N B^*(j)$$

Next period income I_1 is given by

$$(20) \quad \tilde{I}_1 = \tilde{P}Q(X) + \bar{Y} + (1+r)B + \sum_{j=1}^N (1+r^*(j)) B^*(j)$$

where r is the domestic safe interest rate and $r^*(j)$ is the effective interest rate on the foreign bond j , which is uncertain as it includes the effect of movements in exchange rate j . $Q(X)$ is a concave production technology.

The optimal production decision is given by expression (19),

$$(21) \quad (Q'(X))^{-1} = \frac{E(\tilde{P})}{1+r} + \frac{\delta \text{cov}(U'(I_1), \tilde{P})}{U'(I_0)}$$

The optimal amount invested or borrowed in foreign currency j is given by the solutions to the following N implicit expressions,

$$(22) \quad U'(I_0) - \delta E(U'(I_1)E(1+r^*(j))) - \delta \text{cov}(U'(I_1), (1+r^*(j))) = 0 \quad j = 1, \dots, N$$

40. The solution for the optimal amount invested implies that the firm will always invest a lower amount, and subsequently produce less, in the presence of uncertainty about the future domestic currency price of the export good than in absence of export good price uncertainty. Given the concavity of the utility function, the covariance between $U'(I_1)$ and P will always be negative, which, given the concavity of the production function $Q(X)$, implies

that investment and production will be reduced below the certainty solution. Using now the currency composition of the portfolio of the foreign bonds as an active hedging instrument will reduce the conditional uncertainty in next period's income, I_1 , compared to the situation in which the currency composition of external assets was ad-hoc determined. Subsequently, an active hedging strategy using foreign bonds will lead to an increased production of the exportable good.¹⁰

41. For the exact calculation of the optimal portfolio composition we use the special case of a quadratic utility function, $U(I) = aI - (b/2)I^2$. The optimal amounts invested (or borrowed) are:

$$(23) \quad B^* = \left(\frac{a - bI_0}{\delta b} \right) V_{ee}^{-1} (r^* - 1r) - V_{ee}^{-1} V_{es} \begin{pmatrix} Q(X) \\ 0 \end{pmatrix}$$

where the first element in the matrix V_{es} is the domestic currency price of the commodity. The solution is similar to equation (16) and confirms the mean-variance (speculative) portfolio and the income hedge $V_{ee}^{-1} V_{es} [Q(X); 0]$.¹¹ To solve for both the optimal production and portfolio decisions would require the solution of a simultaneous system of $N+1$ equations with $N+1$ unknowns. The portfolio composition decision influences the production decision as it affects the covariance between $U'(I_1)$ and P . Vice-versa, the production decision affects B^* through $Q(X)$. However, the basic nature of the composition of the hedge portfolio remains as in equation (16).

¹⁰ If there are other shocks affecting I_1 , which are negatively correlated with P , then it might be optimal not to hedge the commodity price uncertainty with borrowing (or investing in) a portfolio of foreign bonds.

¹¹ This could be expanded to more goods produced and could include multiple goods consumed.

42. Summarizing, we can state that the models outlined above indicate in a consistent fashion that the optimal external liability composition of a country depends on the following factors:

- a) the shares of consumption expenditures spent on the different goods in the country.
- b) the production structure of the country's economy.
- c) the correlation (covariance) matrix of domestic goods prices and exchange rates.
- d) the correlation (covariance) matrix of exchange rates.
- e) the expected costs of borrowing in each of the foreign currencies.
- f) the correlation (covariance) matrix of the expected costs of borrowing in each of the foreign currencies
- g) the correlation (covariance) matrix of exchange rates and expected receipts and payments in foreign currencies, and
- h) the level of risk tolerance in the country.

The optimal currency composition of external liabilities for a country that only wants to hedge itself against commodity price and exchange rate movements and that does not want to speculate on relative exchange rate movements (i.e., that takes the view that the expected costs of borrowings in each of the foreign currencies are equal) is shown to depend solely on the covariances between commodity prices and exchange rates, the covariances among exchange rates, and the covariances between exchange rates and expected net foreign currency receipts.

V. ESTIMATION OF THE OPTIMAL CURRENCY COMPOSITION OF EXTERNAL DEBT

43. Many articles have reported the results of optimal international asset allocation using portfolio choice models similar to the one we have outlined above. Examples are the articles by Macedo (1982, 1983, 1984), Adler and Dumas (1983), Jorion (1985), Cholerton, Pieraerts and Solnick (1986), Brown, Papell and Rush (1986), Dumas and Jacquillat (1987), and Eun and Resnick (1987). In addition, there exists a large literature documenting empirical research on what one can call the inverse of the optimal portfolio choice--the determination of the forward exchange risk premium in relation to countries' consumption and asset shares. Frankel's (1982) article is a good example of this type of empirical test. Typically, these tests have substituted ex-post means and variances of realized returns for conditional expected means and variances. The optimal portfolios have then been derived without taking into account the uncertainties inherent in the parameter values. As it turns out, the optimal portfolios are very sensitive to the parameter values and thus to the particular period over which these parameters were estimated.¹² As a result the derived portfolios almost always perform very poorly out-of-sample. In addition, the optimal portfolios are very unstable: the proportions allocated to each asset/liability are extremely sensitive to variations in expected returns, and adding a few observations might change the portfolio composition completely. Also, optimal portfolios

¹² Dumas, in his discussion of de Macedo's (1984) article, points out that in a controlled experiment of drawing from a sample with a known distribution, the derived optimal portfolio can easily vary between an investment share of -1 and +1 in the risky asset. Even if we know the estimation risks, there is no known way of translating these estimation risks into resulting reliabilities for the optimal portfolios chosen.

are not necessarily well diversified and often a corner solution appears; where most of the investments are zero and large proportions are assigned to countries with relatively small capital markets, but large average returns.

44. These results suggest close examination of the impact of estimation risk in our model. As it turns out, estimation risk does not present problems in calculating the optimal currency composition of external liabilities for a number of reasons.

45. First, the derived optimal mean-variance portfolios turn out to depend more heavily on the estimates for the expected returns and less on the estimates for the covariance-variances matrices. A high mean return tends to skew the mean-variance portfolio substantially, while a similar change in return has less impact on the estimates for the covariance-variance matrices. If the true processes of commodity prices and exchange rates are lognormal Brownian motions, then the assumption of continuous trading implies that the true covariances and variances can be calculated, while the true means can only be estimated (with a standard deviation that goes to zero as the observation interval--not the number of observations--becomes of infinite length). In the model defined, only the mean-variance portfolio depends on the excess returns on foreign bonds. The other hedging portfolios only depend on the covariance-variance matrices and some parameters. This could imply that the hedging portfolios are less susceptible to estimation risks than the mean-variance portfolio. In addition, the relative weight of the mean-variance portfolio compared to the hedge portfolios is likely to be small, as it seems reasonable to assume that the developing countries are quite risk averse. Developing countries are likely to be more interested in a risk-minimizing portfolio, which consists largely of hedge portfolios, than in a portfolio

that involves some price speculation. As the hedge portfolios do not depend on mean returns, this might improve the out-of-sample performance.

46. Second, several papers on optimal international diversification have successfully employed estimation techniques that rely on Bayesian estimates of means and/or covariances-variances, employing variations of the so-called James-Stein estimators. Examples are Dumas and Jacquillat (1987), Jorion (1985) and Eun and Resnick (1987). The first article presents the out-of-sample results of an international portfolio strategy that is based on the historically estimated covariance-variance matrix of excess returns and on means which are weighted averages of the historically observed means and a common prior expected value. The authors show that the out-of-sample performance of the investment policies that put a very low (even zero) weight on past observations are very satisfactory (often better) compared to the performances of policies which put a large weight on past observations. Investment policies which place a weight of zero on past observations do not place any bet on the future expected conditional evolution of exchange rates as they rely entirely on the neutral common prior. The exact value of a common prior turns out to be immaterial to the portfolio choice. The latter two articles differ from Dumas and Jacquillat's in the sense that their common prior is the grand mean of the historically observed returns. Such a prior hypothesis shrinks the efficient portfolio towards the minimum variance portfolio, which is country specific. Both articles indicate that such a policy was ex-post equally or more successful than classical ones which rely on sample averages. As stated by Jorion: "the merit of diversification rests in lowering risk, rather than pursuing higher returns" [or, for that matter, lower costs]. Developing countries presumably do not always want to take an

explicit view on exchange rate movements and on the associated expected costs of borrowing in a certain currency. It is thus conceivable that developing countries either use in their currency choice a prior which states that all expected costs in their home currency are equal but unknown or else the prior which states that all expected costs are equal to a known value. The three articles discussed indicate that such a liability policy might perform very satisfactorily out-of-sample.

47. Third, since Engle introduced his Autoregressive Conditional Heteroskedasticity (ARCH) estimator, several articles have applied ARCH estimators to the behavior of exchange rates, for instance Diebold and Pauly (1988) and Diebold and Nerlove (1988). ARCH estimators might be able to reduce the estimation risk of the covariance-variance matrices, as they allow for conditional varying variances and covariances.¹³

48. Fourth, even though an instrument whose price is one-to-one related to a measure of volatility is not traded, foreign currency options can provide an implicit estimate of time-varying second moments of exchange rates, which may be used in deriving conditional covariance-variance matrices (see for instance, Lyons (1986)).

49. Fifth, recent research has investigated the relationships between changes in commodity (and other goods) prices, and (nominal) exchange rates (for instance, Giovannini (1986) and Varangis and Duncan (1987)). Most of this research documents fairly stable, significant relationships between changes in exchange rates and domestic commodity prices. In general, the findings are

¹³ Conditional varying variances and covariances might imply a different optimal portfolio choice than the portfolio choice derived under the assumption of constant variances and covariances.

that exchange rate changes are less than completely passed through on domestic commodity prices. The result is that exchange rates will have real effects on the trade flows of commodity exporting developing countries.

VI. POLICY RULES FOR THE CURRENCY COMPOSITION OF EXTERNAL DEBT

50. From the theoretical analysis we are able to extract a set of policy rules which can be used for determining the optimal currency composition of a government's net external liabilities in a small open economy. We assume that the country is only interested in identifying the risk-minimizing hedge portfolio of external liabilities. The country is not assumed to take an active view on exchange rate movements and associated costs of borrowings. We use some of the techniques introduced by Adler and Dumas (1980) and later followed up by others {most recently by Oxelheim and Wihlborg (1987)} for the operational measurement of the economic exposure of a firm to exchange risks.

51. Economic exposure for the firm has been defined in terms of the sensitivity of its objective function with respect to unanticipated changes in exchange rates. The firm's objective function may be defined in terms of the net present value of future expected cash flows, or in terms of near-term cash flows or profits. The operational way of measuring exposure as a cash flow sensitivity, as introduced by Adler and Dumas (1980), is then to measure the covariance between cash flows and the exchange rate relative to the variance of total cash flows, i.e. $\text{cov}(CF_t, e_t)/\text{Var}(CF_t)$. CF_t is here the cash flow in terms of domestic currency in period t and e_t the relevant exchange rate (e.g., Peso/\$) at time t . This exposure measure can be obtained by regressing cash flows on the exchange rate (multiple exposure measures follow if one regresses the cash flows on multiple exchange rates). A related exposure measure is discussed by Oxelheim and Wihlborg (1987). They suggest that a firm trying to evaluate its exposure(-s) runs a multivariate regression which not only includes the exchange rate(-s) but also other (macro-)economic variables

for which one would want to hold the exposure measure(-s) constant. The exposure measure(-s) would then become the partial correlation coefficient(-s) of the cash flows with the exchange rate(-s). In these models the exposure to the exchange rate could be hedged with financial instruments (futures, forwards, swaps, borrowings, etc.) or through adjustments in the sourcing, selling and/or production mixes.

52. Applying this technique to the exposure measurement of a small open economy facing uncertain exchange rates, requires us to assume that the country's objective function can adequately be represented in terms of the expected value and variance of the future cash flows derived from exports and paid on imports.¹⁴ This casts the objective function for the country in terms of the mean and variance of cash flows only instead of in terms of a utility function defined over its consumption of the different goods. In addition, the country has a "neutral" view on exchange rate movements and the associated costs of borrowing in the different currencies. The objective function chosen further implies no speculative portfolios but risk-minimizing portfolios only. Exposure to the different exchange rates and the composition of the risk-minimizing portfolios is estimated as follows.

Step 1: Estimate (OLS) the following equation for each (group of) commodity the country exports, ¹⁵

¹⁴ This assumption is also used in research on the usefulness of commodity futures markets to commodity-exporting nations. The commodity futures markets allow the exporting nations to hedge against commodity price risks and thus to obtain a preferred combination of risk and return. See for instance, Germill (1985).

¹⁵ Healy (1981) uses a similar simple regression technique for the optimal diversification of foreign exchange reserves.

$$(22) \quad \log(P_i^e Q_i^e) = \alpha_i^e + \sum_{j=1}^N \beta_{ij}^e \log(e_j) + \epsilon_i^e \quad i = 1, \dots, H$$

where Q_i^e is the number of units of commodity i exported; $P_i^e Q_i^e$ stands for the domestic currency value of exports of good i ; and e_j , $j = 1, \dots, N$, are the relevant N nominal exchange rates. The specification is in log terms in order to capture the elasticities of the cash flows with respect to the exchange rates.¹⁶ Similar equations are estimated for imports,

$$(23) \quad \log(P_i^m Q_i^m) = \alpha_i^m + \sum_{j=1}^N \beta_{ij}^m \log(e_j) + \epsilon_i^m \quad i = 1, \dots, H$$

Step 2: Calculate the weighted average β_j of the commodity(-group) specific β_{ij} , using as weights the current domestic currency value of exports and imports i , $i = 1, \dots, H$, where the import weights enter negatively.

Step 3: If necessary, rescale the β_j 's such that the weights add up to one. The β_j 's are now the (relative) measures of the aggregate exposure of the country to changes in the exchange rates j . In a risk-minimizing hedge the

¹⁶ It can be shown that the β_{ij} estimated in equation (22) are equal to $\pi_{ij} \times [1 + \epsilon_{ij}] + 1$, where π_{ij} is (a variant of) the pass-through elasticity, $\pi_{ij} = (\delta P_{ij} / \delta e_j) \times (e_j / P_{ij})$, P_{ij} is the foreign currency price of good i in country j , and ϵ_{ij} is the demand elasticity in country j for good i . The exact exposure of the country to exchange rate changes depends now on the magnitude of the pass-through effect and foreign demand elasticities. With no pass-through the exposure to the exchange rate j is $+1$; with complete pass-through ($\pi_{ij} = -1$) it is $-\epsilon_{ij}$. This approach is partial equilibrium and ignores for instance supply effects, which would depend on the elasticity of marginal cost with respect to output as shown in Varangis and Duncan (1987). It is meant for illustrative purposes.

β_j 's will equal the relative shares of foreign currency j in the optimal external liability portfolio of the country.¹⁷

53. The results so far are "ex-post": i.e., the minimum-risk portfolio is calculated using historical data which could be compared to a naive strategy of hedging based on nominally denominated trade flows over the whole sample period. In reality, the country would only have had information up to time t with which to design the hedges for time $t+1$. To show the effectiveness of ex-ante risk-minimization one can construct the portfolios for period $t+1$ that are based on a sample of observations up to time t and compare the outcome of these hedges to the outcome of an alternative strategy. The sample is then updated to include time $t+1$ information, the portfolios are constructed for period $t+2$, and compared to the outcome of the alternative strategy in period $t+2$. The effectiveness of ex-ante hedges can be indicated by calculating the risk-reduction realized over the whole hedging period relative to the costs (or, returns) realized.

¹⁷ It can be shown that the optimal shares in the risk-minimizing portfolio are the regression coefficients β_{ij} . The formula for the vector of OLS coefficients in the regression of equation (22) and (23) is $\beta = (X'X)^{-1} X'c$, where X is the observation matrix for the exchange rate variables and c is the vector of time-series observations on the cash flows. The term $(X'X)$ is an estimate of the variance-covariance matrix of the (logarithms of the) exchange rates and $X'c$ is a vector whose elements are the covariances of (the logarithms of) each exchange rate with the cash flows expressed in domestic currency. The expression of the vector of regression coefficients β is thus similar to the expression for the portfolios that serve as price hedges in equations (8) and (16).

VII. COMPUTATION

54. The analysis so far has investigated the optimal foreign currency composition of the country's portfolio, which includes private net assets, central bank exchange reserves and government foreign debt. In applying an optimal currency composition model one would take both the currency composition and the share in national wealth of private net foreign assets as given. The optimal foreign currency composition is then determined in terms of the currency composition of the net portfolio of central bank reserves and government foreign debt. Translating the optimal currency composition of the country into an optimal government currency composition should not be a problem as it simply requires subtracting the currency composition of private net foreign assets weighted by its share in national wealth, from the country's optimal foreign currency composition [see also Svensson (1987)].

VIII. APPLICATIONS

55. The model outlined above is currently being estimated for a large developing country that has been adversely affected by recent exchange rate and commodity price movements. This application is part of a theoretical as well as practical study the Bank is currently undertaking on currency management of external debt.

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